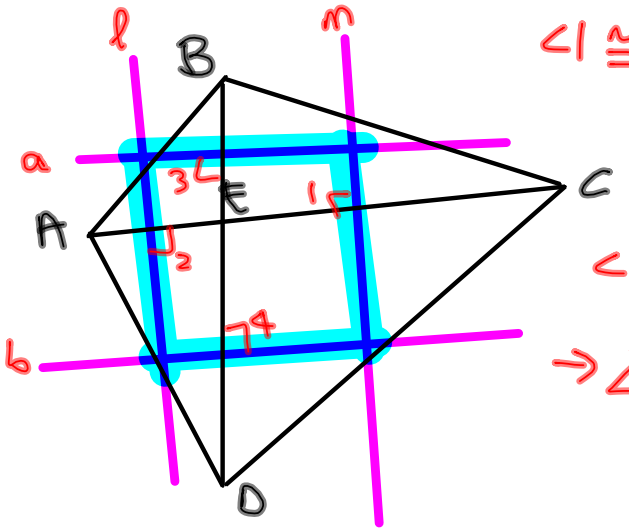
8.3.7★ The diagonals of convex quadrilateral $ABCD$ meet at E. Prove that the centers of the circumcircles of $\triangle ABE$, $\triangle BCE$, $\triangle CDE$, and $\triangle DAE$ are the vertices of a parallelogram. **Hints:** 80



$\angle 1 \cong \angle 2 \rightarrow \ell \parallel m$

$\angle 3 \cong \angle 4 \rightarrow a \parallel b$

$\rightarrow \square P$

8.6.6 Point E is inside square $ABCD$ such that $\triangle ABE$ is equilateral. Given that $AB = 4$, find the following:

(a) AE. $= 4$

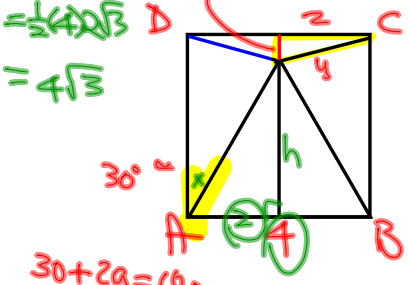
(b) $[ABCD]$. $= 16$

(c) $[ABE]$.

(d) $\angle DAE$ and $\angle DEA$.

(e) The area inside $ABCD$ but outside $\triangle ABE$. $16 - 4\sqrt{3}$

(f) CE.



$\frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3}$

$4 - 2\sqrt{3}$

$2^2 + h^2 = 4^2$

$h^2 = 12$

$h = \sqrt{12} = 2\sqrt{3}$

$30 + 2a = 180$

$a = 75°$

$(4-2\sqrt{3})(4-2\sqrt{3}) = 16 - 8\sqrt{3} - 8\sqrt{3} + 12$

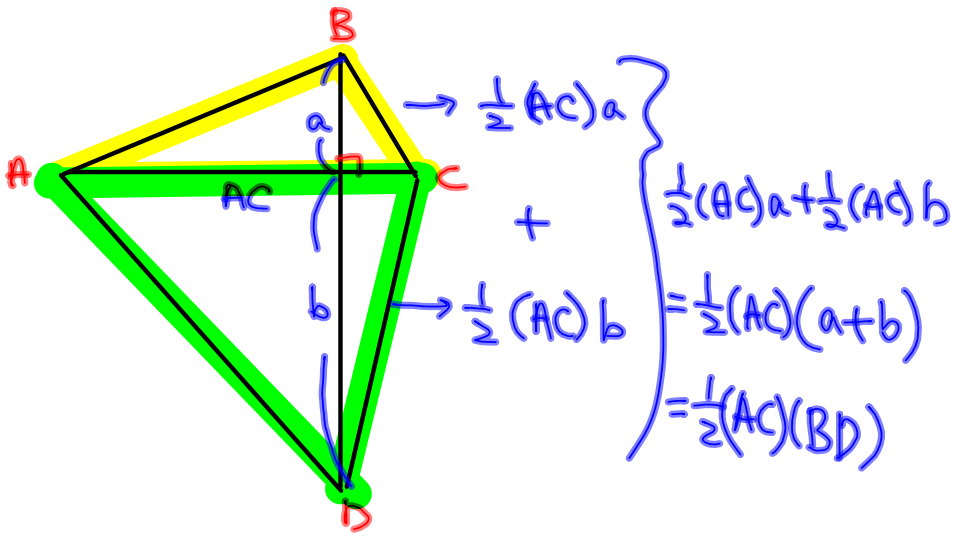
$2^2 + (4-2\sqrt{3})^2 = y^2$

$4 + 28 - 16\sqrt{3} = y^2$

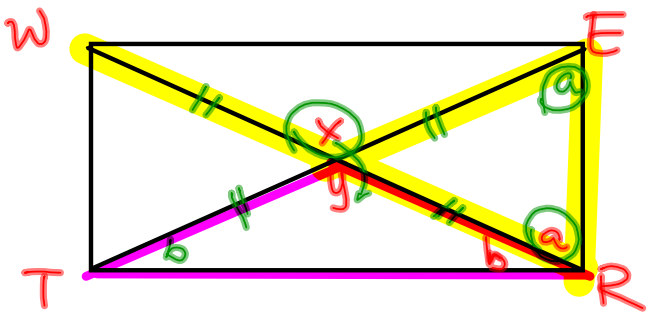
$32 - 16\sqrt{3} = y^2$

$\sqrt{32 - 16\sqrt{3}} = y$

8.4.3 Diagonals $\overline{AC}$ and $\overline{BD}$ of quadrilateral $ABCD$ are perpendicular. Prove that $[ABCD] = (AC)(BD)/2$.
Hints: 438



8.5.3 Diagonals $\overline{WR}$ and $\overline{ET}$ of rectangle $WERT$ meet at Y. Given that $\angle WYE = x$, find $\angle ERY$ and $\angle YRT$ in terms of x.



$x = 2a$

$a = \frac{x}{2}$

$2b + x = 180$

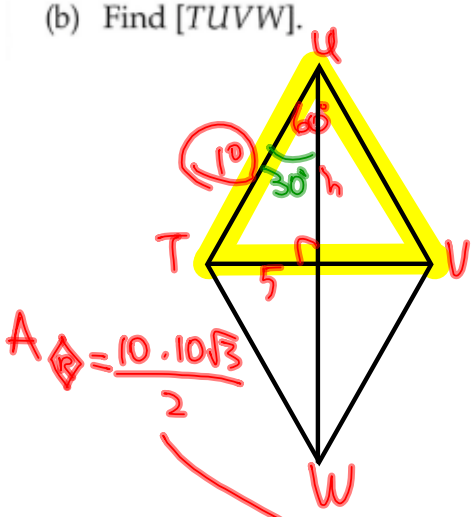
$2b = 180 - x$

$b = 90 - \frac{x}{2}$

8.4.4 $TUVW$ is a rhombus with $TU = 10$ and $\angle TUV = 60°$.

(a) Show that $\angle TUW = 30°$.

(b) Find $[TUVW]$.



$5^2 + h^2 = 10^2$

$h^2 = 75$

$h = \sqrt{75} = 5\sqrt{3}$

$A_{\triangle} = \frac{10 \cdot 10\sqrt{3}}{2}$

$\triangle = \frac{5 \cdot 5\sqrt{3}}{2} = \frac{25\sqrt{3}}{2}$

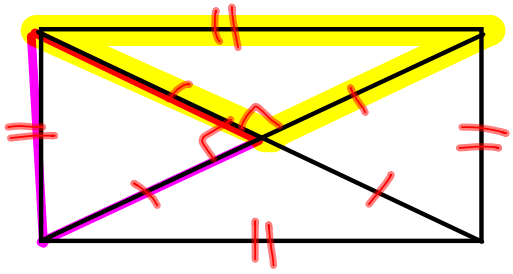
$\diamond = 4\left(\frac{25\sqrt{3}}{2}\right) = 50\sqrt{3}$

8.6.4 Show that a rectangle with perpendicular diagonals must be a square.

$= \angle$'s

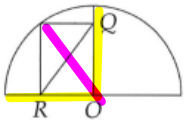
$=$ sides

$=$ $\angle$'s



$2 \cong \triangle$'s

8.5.4 A semicircle with center O has a radius of 9 cm. What is the number of centimeters in the length of $\overline{RQ}$, a diagonal of the rectangle shown? *(Source: MATHCOUNTS)*



9.